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Theoretische Informatik (Kernfach) *SS 2004*
Exercise Set 4

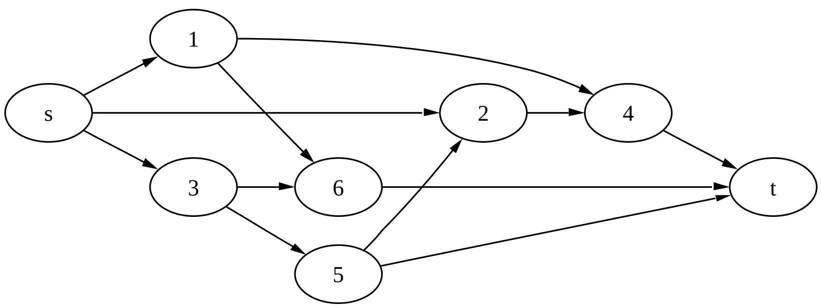
Exercise 1

(a) Prove Lemma 1.12 in the hand-out *Network Flows*.

(b) Show that for any Δ the capacity scaling algorithm makes no more than $2m$ augmentations in $G_f(\Delta)$.

Exercise 2

Consider the following network:



All edge-capacities are 1.

Simulate the shortest augmenting path variant of the Ford-Fulkerson algorithm on this network. In particular write down the residual network and the current flow in every step. Start with the path $s \to 1 \to 4 \to t$.

Exercise 3

Show: For every network with m edges there is a sequence of at most m augmentations which produces a maximum flow.

Exercise 4

An $n \times n$ grid is an undirected graph consisting of n rows and n columns of vertices. (See Figure 1.) We denote the vertices in the ith row and the jth column by (i, j). All vertices in a grid have exactly four neighbors, except for the boundary vertices, which are the points (i, j) for which $i = 1$, $i = n$, $j = 1$, or $j = n$.

Given $m \leq n^2$ distinct starting points $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$ in the grid, the escape problem is to determine whether or not there are m vertex-disjoint paths from the starting points to any m different points on the boundary. (See for example Figure 1.)

(a) Consider a flow network in which vertices, as well as edges, have capacities. That is, the positive net flow entering any given vertex is subject to a capacity constraint. Show that determining the maximum flow in a network with edge and vertex capacities can be reduced to an ordinary maximum flow problem on a flow network of comparable size.

(b) Describe an efficient algorithm to solve the escape problem.
(*Hint:* You might consider part (a).)

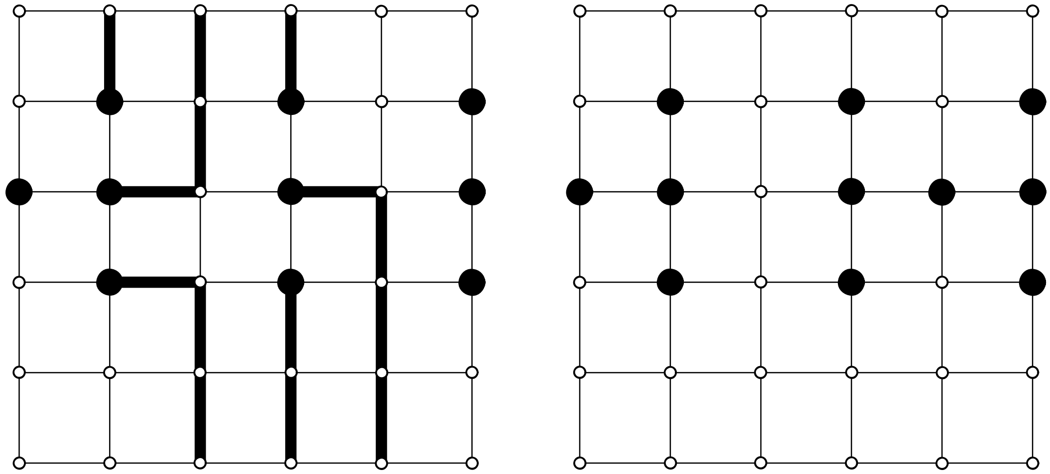


Figure 1: Two grids for the escape problem. Starting points are black, all other grid points white. In the left configuration an escape is possible whereas in the right one there is no escape.

Due-date:22.04.04 in the lecture