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Theoretische Informatik (Kernfach) *SS 2004*
Exercise Set 9

Exercise 1

Let $G = (V, E)$ be a connected undirected graph and let $s, t \in V$ be vertices. Consider the following linear program with variables x_v, $v \in V$: maximize x_t subject to $x_s \leq 0$ and $x_v \leq x_u + 1$ for every edge $\{u, v\} \in E$. Show that this linear program always has an integral optimal solution. What is the meaning of the optimal solution?

Exercise 2

Let $G = (V, E)$ be a connected undirected graph and let $s, t \in V$ be vertices. Let us consider the following integer program with variables x_e, $e \in E$, and y_v, $v \in V$: minimize $\sum_{e \in E} x_e$, subject to $x_e \in \{0, 1\}$ for every $e \in E$, $y_v \in \{0, 1\}$ for every $v \in V$, $y_s = 0$, $y_t = 1$, and $y_v \leq y_u + x_{\{u,v\}}$ for every edge $\{u, v\} \in E$. Explain how an optimal solution corresponds to a minimum edge cut separating s and t.

Exercise 3

Let $G = (V, E)$ be a connected undirected graph. We are given the following integer program with variables $x_e \in \{0, 1\}$ for every $e \in E$: maximize $\sum_{e \in E} x_e$, subject to $x_e \in \{0, 1\}$ for every $e \in E$ and $\sum_{e \ni v} x_e = 2$ for every $v \in V$. Does every optimal solution correspond to a Hamilton cycle (a cycle passing through all vertices of G)?

Exercise 4

Consider the linear relaxation of the maximum matching problem on a bipartite graph as in the lecture notes. Using results about network flows learned earlier, prove that it always has an integral (i.e. zero-one) optimal solution.

Exercise 5

Let $\mathcal{F}$ be a system of m subsets of the set $X = \{1, 2, \ldots, n\}$. Prove that each of the points of X can be colored red or blue in such a way that for every set F of $\mathcal{F}$, the number of red points in F differs from the number of blue points in F by at most $\sqrt{2n \ln(2m)}$.
Hint: Use a random coloring and the Chernoff inequality.

Due-date: 03.06.04 in the lecture