ETH

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Algorithms, Probability, and Computing   Exercises KW47   HS16

The following exercises will be discussed in the exercise class on November 23, 2016. Please hand in your solutions not later than November 22.

Exercise 1: Solving Linear Programs via Binary Search

Work out the idea from the lecture of using an algorithm for the feasibility problem for solving linear programs, based on the idea of binary search for the optimum value. Theorem 6.2 is useful for analyzing the number of steps of the binary search.

Exercise 2: Equivalence of the Three Farkas Lemmas

(a) Explain why the $\mathbf{y}$ as in Lemma 6.5 indeed certifies the nonexistence of a nonnegative solution.

(b) Prove that all of the three variants of the Farkas lemma, I–III, are mutually equivalent. (You may want to look at last week's exercise about the equational form for inspiration.)

Exercise 3: Deciding Feasibility vs. Finding Feasible Solutions

Suppose that we have an oracle that, given a system of linear inequalities, decides its feasibility (outputs YES or NO). Design an algorithm that computes a solution of a given system of linear equations and inequalities, provided that one exists, in polynomial time and with polynomially many calls of the oracle.

(a) How can we proceed if there are only equations in the system?

(b) If there is at least one inequality, use the oracle to check if there is a solution satisfying that inequality with equality, and take appropriate actions depending on the outcome.