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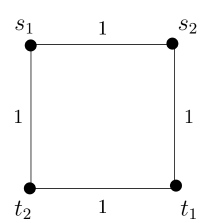
Algorithmic Game Theory FS07

Exercise sheet 10

First, we shortly describe the class of *Global Connection Games*. An instance of such a game is specified by a weighted directed graph $G = (V, E)$, where every edge e is assigned a weight (cost) $c_e \geq 0$, and k pairs (s_i, t_i), where s_i and t_i represent the *source* and the *destination* of a player i, $i = 1, \ldots, k$, respectively. A strategy of player i is a directed $s_i - t_i$-path P_i.
Goal of every player is to minimize own cost to connect the source s_i to the destination t_i. The cost depends on the *cost-sharing scheme* that we choose to apply to split the cost of commonly used edges, i.e., edges that are part of the paths of different players. Often we will use the *Fair* (or *Shapley*) cost-sharing scheme that splits evenly the cost of an edge among the players using it. The *social cost* of a solution is the sum of the cost charged to the players.
Once more, a *Nash Equilibrium* is a strategy profile, where no player can unilaterally strictly decrease the own cost by changing strategy.

EXERCISE 10.1:
In the lecture it has been shown that the Global Connection Game with the cost-sharing scheme, where exactly one player is charged the total cost of one edge regardless of how many players are using it, might not have a Nash Equilibrium. The example that demonstrates this for undirected graphs is depicted below.



a) Construct an example with a directed graph that shows that Nash Equilibria do not necessarily exist when applying the aforementioned cost-sharing scheme.

b) The previous point states that a Nash Equilibrium is not guaranteed to exist. Does this contradict Nash Theorem about the existence of Nash equilibrium? Justify your answer.

EXERCISE 10.2:
Prove that in any Global Connection Game (with fair cost-sharing scheme) with k players, the Price of Anarchy is at most k.

EXERCISE 10.3:
The lower bound construction presented in the lecture showing that for any $\varepsilon > 0$ the Price of Stability is at least $H_k/(1+\varepsilon)$ (H_k is the k-th harmonic number) uses weights on the edges (edges with different costs). Analyse the lower bound for unweighted networks, where each edge has unit weight.

Deadline. You are to hand in your solutions during the lecture on Thursday, December 13th, 8:15-10:00 in CAB G11.