Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Federal Institute of Technology at Zurich

Institut für Theoretische Informatik
Matúš Mihalák
Peter Widmayer
Davide Bilò
Elias Vicari

Algorithmic Game Theory FS07

Exercise sheet 2

EXERCISE 2.1:
Consider two players, Colin and Rose, in a tennis match. Colin is about to serve and should decide whether to aim to Rose's forehand or backhand. Meanwhile, as Colin's serve is usually very good, Rose should decide whether to move to the forehand or backhand side, trying to anticipate the serve. If Rose's guess is right, she increases her chances to eventually win the rally.

The situation can be modelled as a bimatrix game, where the following table represents every player's chances to win the rally.

Rose \ Colin	F	B
F	$(90, 10)$	$(20, 80)$
B	$(30, 70)$	$(60, 40)$

Clearly, if Colin (the serving player) plays repeatedly a pure strategy, he is going to lose the match. Can a mixed strategy save the Colin's game?

a) Compute the Colin and Rose's expected payoff if they both play F or B with 50% probability.

b) Compute the Nash Equilibrium of this game.

EXERCISE 2.2:
Consider the following strategic game and compute all its Nash Equilibria.

Rose \ Colin	A	B	C
A	$(0, 0)$	$(3, 0)$	$(0, 2)$
B	$(0, 3)$	$(0, 0)$	$(3, 2)$
C	$(2, 0)$	$(2, 3)$	$(2, 2)$

EXERCISE 2.3:
Consider a strategic game with 2 players, where each player has $n \geq 2$ strategies. Show that if the payoffs for this game are drawn independent at random with a uniform distribution over $[0, 1]$, the game admits a pure strategy Nash Equilibrium with probability at least $1 - e^{-1}$.

Hint: Recall that $\left(1 - 1/x\right)^x \leq e^{-1}$ for all $x \geq 1$.

EXERCISE 2.4: Passenger and Inspector (A modification of the game from the lecture of Bernhard von Stengel at LSE, London)
A passenger uses a public transport, but selfishly he is not sure whether buying the ticket all the time is the best strategy for his finances. The passenger is thus weighing two possible strategies: to buy a ticket, or not to buy a ticket. The inspector of the public transport company has also two strategies: to inspect or not. The ticket costs 1 franc. Every inspection incurs a cost of 2 franc to the inspector. If the inspector catches someone without a ticket, on top of the 2 franc it costs her 1 more franc for administrative costs. The passenger without a ticket then has to pay the regular fare fee and a fine of 80 francs. The total 80 francs counts as passenger's lost, but not as the inspector's win, as the 80 francs go entirely to the company who runs the public transport. If the passenger does not have a ticket, the inspector would prefer to inspect, as then the passenger has to pay also the fare fee of 7 francs to the inspector, which is a part of his salary.

a) Model the game as a matrix game.

b) Compute a Nash equilibrium of the game.

c) What is the expected payoff to the passenger and the inspector in your equilibrium?

d) What happens if the company decides to increase the fine from 80 to 160 francs?

EXERCISE 2.5: (from the lecture of Bernhard von Stengel, LSE London)
Consider the following strategic game.

		Colin				
		a	b	c	d	e
Rose	T	$(0,2)$	$(2,4)$	$(1,3)$	$(0,5)$	$(3,4)$
	B	$(1,7)$	$(0,4)$	$(4,5)$	$(1,0)$	$(0,8)$

a) Suppose that Rose plays strategy T with probability p and strategy B with probability $1-p$. Draw the expected payoffs to Colin for all her strategies a, b, c, d, and e as a function of the probability p.

b) Find all pure or mixed equilibria of the game.

Deadline. You are to hand in your solutions during the lecture on Thursday, October 18th, 8:15-10:00 in CAB G11.