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# **Algorithmic Game Theory** FS07

Exercise sheet 4

**EXERCISE 4.1:**

VCG mechanisms have nice theoretical properties, but they are not widely used in practise because of several drawbacks. Goal of this exercise is to point out two of these. We focus on second-price auctions, which are an application of VCG mechanisms in combinatorial auctions.

a) Construct an instance for a second-price auction, where the following occurs: the bidders have arbitrarily large interest in (at least) a subset of the items, yet the total revenue for the seller is $\varepsilon$, for any $\varepsilon > 0$.
**Hint:** It is possible to construct such an example with two items and three bidders.

b) Show that the second-price auction is not monotone, i.e., the seller's revenue can strictly decrease if new bidders join the auction.

**EXERCISE 4.2:**

a) Consider the *third-price* auction: in a single-item auction with at least three bidders, the item is awarded to the highest bidder at the cost corresponding to the third highest bid. Is this mechanism truthful?

b) Consider the following scenario: in an auction with $n$ bidders, a seller would like to sell $k$ identical items, where $n > k$. Every bidder is interested in a single item and declare a bid for it. Design a truthful mechanism for this auction.

**EXERCISE 4.3:**

Recall that a mechanism is called VCG, if:

i) it optimizes a utilitarian objective function, where the outcome is

$$o \in \arg_o \max \sum_{i=1}^{n} v_i(o, a_i).$$

Recall that $v_i(o, t_i)$ is the valuation of player $i$ with type $t_i$, and $a_i$ is the value reported by player $i$;

ii) the payment to player $i$ is of the form:

$$p_i = h(a_{-i}) - \sum_{j \neq i} v_j(o, a_j)$$

for an arbitrary function $h$.

Prove that any VCG mechanism is truthful.

**EXERCISE 4.4:**
Let $G = (V, E)$ be a 2-edge-connected graph, i.e., between any two nodes of the graph there exist at least two edge-disjoint paths.
Suppose that $G$ is a network and that every edge (link) $e$ is owned and operated by an agent $A_e$. A company is interested in buying a possibly cheap subset $T$ of the links, such that every pair of nodes of $G$ can communicate along edges of $T$. Hence we assume that $T$ induces a *spanning tree* of $G$. The company asks singularly every agent $A_e$ for the cost $t_e$ they incur in operating the link, which is a private information. The agent can possibly lie about the cost and report $c_e$ to achieve a better utility, which is defined as $p_e - t_e$, if the link joins $T$ and the agent is consequently awarded of $p_e$ francs, or 0, otherwise.

a) Design a mechanism that induces the agents to truthfully announce their cost. The description of the mechanism reports how the links are selected and which payment the agents are awarded.

b) Does a truthful Archer-Tardos mechanism for this problem exist? Justify your answer.

**Deadline.** You are to hand in your solutions during the lecture on Thursday, November 1st, 8:15-10:00 in CAB G11.