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Algorithmic Game Theory FS07

Exercise sheet 5

EXERCISE 5.1:
In the lecture we focused on the computational tractability of the *Allocation problem* (the problem to assign items to buyers, so to maximize the sum of the valuations) for various auction models. It has been shown that in many cases this problem does not admit a polynomial time algorithm (unless $P = NP$). In this exercise we want to discuss two auctions that do admit a polynomial time algorithm (in the number n of players and m of goods). Notice that since the Allocation problem is merely a function that assigns goods to players according to their bids, we do not require the players to tell the truth.

a) Show that the Allocation problem can be solved in polynomial time, if the bidders are single-minded and if the Player i's bid is (S_i, α_i), where $|S_i| = 2$, and α_i is the price that she is willing to pay for every superset of S_i, for every i.
Hint: A *Maximum Weight Matching* can be computed in polynomial time.

b) The direction of a congress hall wants to prepare a schedule of the events to be held in the hall in the next m days. Many associations are interested in renting the congress hall to hold their conferences. Obviously, these associations are willing to rent the hall during *consecutive* days, and there might be overlapping periods among the associations' schedules. The congress hall direction tries to fairly assign the daily slots by organising an auction among the associations. Every organisation submits a sealed bid of the form (S_i, α_i), where $S_i = \{f_i, \ldots, e_i\}$, $1 \leq f_i \leq e_i \leq m$ are the (consecutive) days where association i is interested in the usage of the hall, and α_i is the price they are willing to pay for them $(i = 1, \ldots, n)$.

   Show that the Allocation problem can be solved in polynomial time in this case.
   Hint: Try a dynamic programming approach.

EXERCISE 5.2:
Consider the following two properties for combinatorial auction mechanisms.

i) **Monotonicity.** If a bidder wins with a bid (S, α), she keeps winning if she bids (S', α'), for $\alpha' \geq \alpha$ and $S' \subseteq S$ and the other bids are fixed.

ii) **Critical Payment.** A bidder who wins S pays the minimum value needed for winning: the infimum of all values v' such that (S, v') still wins (given that other bids are fixed).

We are ready to consider the following claims.

a) Prove that a mechanism for single-minded bidders in which losers pay 0 is truthful if and only if the mechanism satisfies the Monotonicity and Critical Payment properties.

b) Prove that the generalized Vickrey Auction for single-minded bidders is truthful using a).

c) Prove that the third-price auction, where the players are interested in a single item, is not truthful using a).

EXERCISE 5.3:
Consider the following online version of Vickrey Auction, where the bidders arrive one by one. The n bidders arrive one at a time; when a bidder shows up, it presents a nonnegative bid; the auction decides whether or not to sell the good to the bidder, and at what price (contrained above by the bid); then the bidder departs, never to return. Assume that the bidders always bid with their true valuations.

Prove that if the valuations of bidders and the order in which they arrive are arbitrary, then no auction can guarantee a near optimum social welfare (the sum of the players' valuations), or in other words, show that for every auction, the social welfare can be arbitrarily smaller than the social welfare of an auction that knows the bidders' valuations and their order of arrival in advance.

Deadline. You are to hand in your solutions during the lecture on Thursday, November 8th, 8:15-10:00 in CAB G11.