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Algorithmic Game Theory FS07

Exercise sheet 8

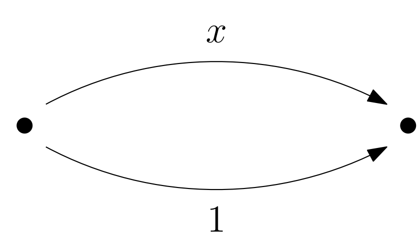
We consider the selfish routing problem from the lecture. Recall some of the notation. The network is given as a directed graph $G = (V, E)$. There are k source-destination pairs (s_i, t_i) with (flow) rates r_i $(i = 1, \ldots, k)$. Each edge e is assigned a latency function $l_e : \mathbb{R}^+ \to \mathbb{R}^+$, which is continuous and non-decreasing. The goal is to send a flow of rate r_i from s_i to t_i. Flow f_e routed through edge e incurs a delay $l_e(f_e)$ per flow unit. We assume that the flow is controlled by infinitely many (selfish) agents. Each agent is interested in her individual delay, i.e., the delay that is incurred by her flow—the sum of edge-delays $l_e(f_e)$ for every edge e that the agent's flow traverses. For flow $f = \{f_e\}_{e \in E}$ (where flow f_e is assigned to edge e), the cost of the flow is $C(f) = \sum_{e \in E}(l_e(f_e) \cdot f_e)$ (i.e., the cost is the total delay incurred by the flow). For the overall performance of the network we are interested in the flow f that minimizes the delay of the flow, i.e., $C(f)$. Such a flow is called an optimum flow. Seeing the setting in the game-theoretic framework, the agents end up routing their flow in an equilibrium (that is called a Nash flow). The price of anarchy is the ratio of the cost of the worst Nash flow and the cost of the optimum flow.

EXERCISE 8.1:
Show that the price of anarchy is one when the latency function of an edge is a monomial of a fixed degree with edge-dependent coefficients: $l_e(f_e) = a_e f_e^d$, for all $e \in E$.

Hint. Establish characterizations for the optimal flow and the Nash flow with this class of latency functions. Proceed in a similar way like in the lecture.

EXERCISE 8.2:
The so called Pigou's example (below) is the famous simple example that shows that the price of anarchy is at least 4/3. We achieve this by setting the upper link to have a constant latency function 1 and the lower link to have a linear latency x. Consider a situation where no constant function can be used as a latency function, but only the continuous non-decreasing c, for which $c(0) = 1$ and the identity function $c(x) = x$ can. Can you modify the Pigou's example and come arbitrarily close to the ratio 4/3 also using this class of functions?

Hint. Use more parallel edges.



Deadline. You are to hand in your solutions during the lecture on Thursday, November 29th, 8:15-10:00 in CAB G11.