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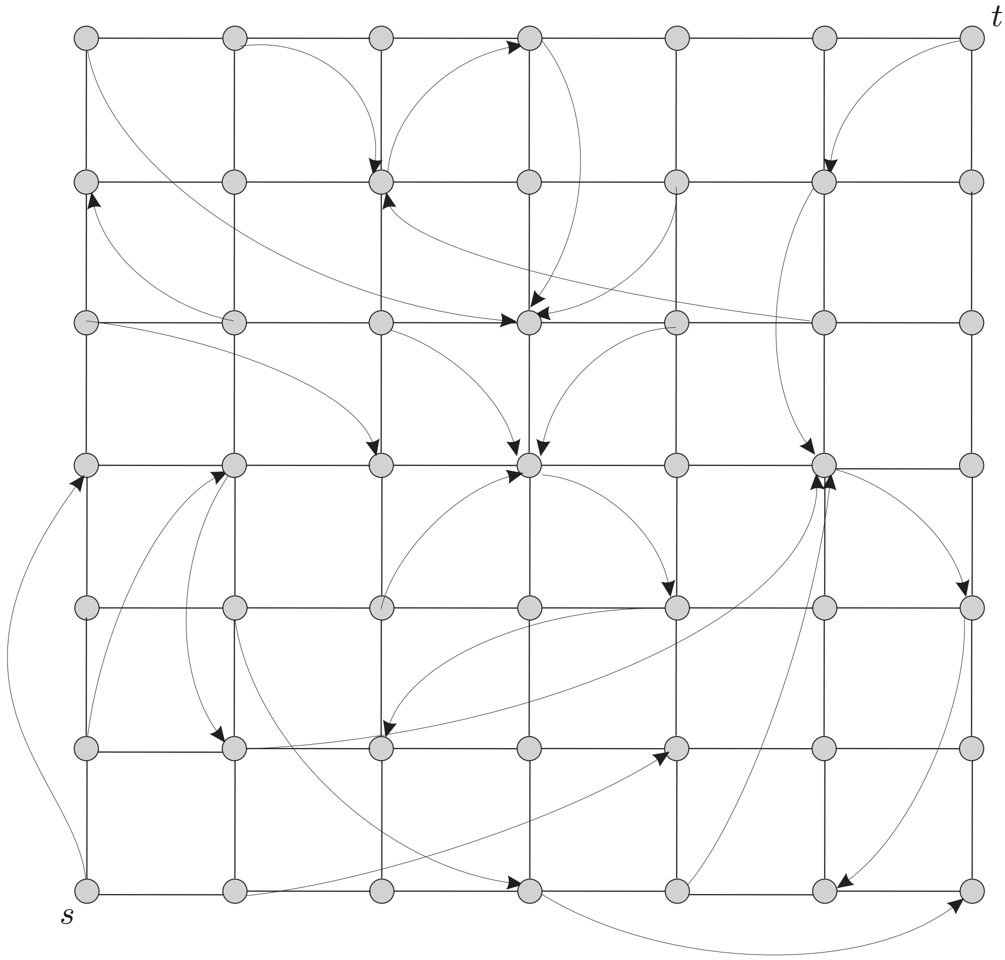
Web Algorithms WS05/06

Exercise sheet 1

EXERCISE 1.1:

At any current node u, the GREEDY algorithm forwards the message to the neighbor of u that has smallest Manhattan distance to the target t. The algorithm LOOK2 is an extension of GREEDY, and uses a lookahead of 2 steps as follows. At a current node u, LOOK2 considers the neighbors of u, as well as their long-range contacts. We call this set of nodes $N^2(u)$. From node u, algorithm LOOK2 forwards the message to the node in $N^2(u)$ that has smallest Manhattan distance to t. Note that this may require traversing two arcs.



A. In the Small World Network (SWN) above, all local contacts (grid arcs) are bi-directed, and the long-range contacts are only shown when different from the local contacts. Construct the $s - t$ path for LOOK2.

B. What is the expected number of steps for LOOK2?

The algorithm LOOKlog has a lookahead of $\log n$, and considers the $\log n$ closest neighbors of the current node u, as well as their long-range contacts. We call this set $N^{\log}(u)$. From node u, algorithm LOOKlog forwards the message to the node in $N^{\log}(u)$ that has smallest Manhattan distance to t, which may again require traversing more than one arc.

C. What is the expected number of steps for LOOKlog?

EXERCISE 1.2:

Consider the SWN for a 3-dimensional grid, where each node u has its 6 neighbors as local contacts, and long-range contacts are generated with the proportional long-range contact probabilities

$$\Pr[u \text{ has long-range contact } v] \sim d(u,v)^{-3}.$$

A. Show that the normalizing constant $\sum_w d(u,w)^{-3}$ for this probability distribution is at most $k_1 \cdot \ln(k_2 n)$, for some constants k_1, k_2.

B. How does the expected number of steps of GREEDY for the 3-dimensional grid compare to the 2-dimensional case?

EXERCISE 1.3:

For the 2-dimensional grid SWN, consider the more general long-range contact probability distribution

$$\Pr[u \text{ has long-range contact } v] \sim d(u,v)^{-r},$$

where $r = 2$ for the SWN discussed in class.
When $0 \leq r < 2$, there exists no decentralized algorithm that constructs paths of length $O(\log^k n)$, for some constant k. The same holds for $r > 2$. Give a brief intuitive explanation for this behavior, for both $0 \leq r < 2$ and $r > 2$.

Deadline: You are to hand-in your solutions during the exercise class on Wednesday, November 2nd, 14.00-15.00 in CAB G 51.