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Exercises for the course

Web Algorithms

Semester WS05/06

Exercise sheet 9

EXERCISE 9.1:

Specify a bidding language (with phantom goods if necessary) such that the following valuations can be expressed:

- A bidder has a valuation v_j for each item j, and values each set of items as the sum of the items valuation, i .e. $v(S) = \sum_{j \in S} v_j$. In addition, each bidder has an upper limit k of total items he wants to buy.
- There are n identical items to sell. Each bidder i has a price $p_{i,j}$ specifying how much he is willing to pay for the j'th item won. The prices $p_{i,j}$ are decreasing for all bidders, that is, $p_{i,1} \geq p_{i,2} \geq \ldots$. Consider the case where each bidder wants to buy at most k items, and the case where each bidder wants to buy at least k items.

EXERCISE 9.2:

Consider the combinatorial auction (CA) with single-minded bidders from the lecture. Let (S_i, b_i) denote the fact that bidder i is willing to pay price b_i for bundle S_i and nothing otherwise. Show that a mechanism fulfilling the four properties exactness, monotonicity, no fee, and critcal value is truthful.

Hint: Show first that bidding for a different bundle S_i' does not yield a higher utility. Next, show that bidding for the true bundle S_i with a false bid b_i' does not yield a higher utility either.

EXERCISE 9.3:

Consider the cautious greedy heuristic for the CA with single-minded bidders from the lecture (where m is the total number of goods in the auction).

- Provide an instance where the allocation of this mechanism (with norm $\|S\| = |S|$) is an m-approximation for the optimal allocation.
- Show that the allocation of this mechanism (with norm $\|S\| = \sqrt{|S|}$) is a $\sqrt{m}$-approximation for the optimal allocation.

EXERCISE 9.4:

Provide a problem instance for each of the following settings:

- The total payments of the Generalized Vickrey Auction (GVA) are higher that the payments of the Cautious Greedy Heuristic (CGH) in a CA with single-minded bidders.
- The total payments of the CGH are higher than the payments of the GVA in a CA with single-minded bidders.
- In a CA with double-minded bidders each bidder i has a valuation given by $(S_{i,1}, b_{i,1}) XOR (S_{i,2}, b_{i,2})$. The cautious greedy heuristic (CGH) can be applied to such bidders by treating the two desired sets as separate bids (but if a bid wins, then the other bid of the same bidder is discarded). Give an example showing that the adapted CGH does not remain truthful for double-minded bidders.

Deadline: You are to hand in your solutions during the exercise class on Wednesday, January 11th, 14.15–15.00 in CAB G 51.