ETH
Eidgenössische Technische Hochschule Zürich
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Algorithms, Probability, and Computing **Exercises KW42** **HS20**

General rules for solving exercises

- When handing in your solutions, please write your exercise group on the front sheet:

  Group A: Wed 14–16

  Group B: Wed 14–16

  Group C: Wed 16–18

  Group D: Wed 16–18

- This is a theory course, which means: if an exercise does not explicitly say "you do not need to prove your answer", then a formal proof is **always** required.

The following exercises will be discussed in the exercise class on October 14, 2020. Please hand in your solutions via Moodle, no later than 10 pm October 11.

Exercise 1

For a permutation π on the keys $\{1..n\}$, let T_π be the search tree that we obtain from inserting all n keys, one after the other, in the order given by π.

Prove: If π is drawn uniformly at random, then T_π has the same distribution as $\tilde{\mathcal{B}}_{[n]}$ from the lecture.

Exercise 2

Suppose you are given a finite set $S \subseteq \mathbf{R}$, $2 \leq |S|$, which is to be preprocessed so that for query $q \in \mathbf{R}$ the answer is 'the' set $\{b_1, b_2\} \subseteq S$ of the two closest numbers in S (i.e. $\max\{|b_1 - q|, |b_2 - q|\} \leq \min_{a \in S \setminus \{b_1, b_2\}} |a - q|$). Follow the locus approach for the problem and describe the resulting partition of regions of equal answers (and be aware of the ambiguity issue, i.e. the 'the' has to be taken with caution).

Exercise 3

Given a sorted sequence $a_0 < a_1 < \cdots < a_{n-1}$ of n real numbers, we consider the convex polygon C with vertices $\left((a_i, a_i^2)\right)_{i=0}^{n-1}$. For $k \in \mathbf{R}$, show that the line with equation $y = 2kx - k^2$ intersects C iff $k \in \{a_0, a_1, \ldots, a_{n-1}\}$.

REMARK: This exercise is supposed to exhibit that deciding whether a line intersects a convex polygon cannot be easier than deciding whether a query key is in a given set of keys.