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Computational Geometry — Exercise Set 2 — HS09

URL: http://www.ti.inf.ethz.ch/ew/courses/CG09/

Exercises

Every week we will hand out an exercise sheet with a complementary material to the lecture. You are advised to solve them and may hand them in to the assistant for corrections and suggestions.

There will be four special series of exercises, which will be obligatory and graded. Three best grades from exercises will contribute to the final grade 10% each.

Exam

There will be an oral exam of 30 minutes during the examination period. Your final grade consists to 70% of the grade for the exam and to 30% of the grades for the exercises.

Exercise 1

A set $S \subset \mathbb{R}^d$ is *star-shaped* $\iff$ there exists a point $c \in S$, such that for every point $p \in S$ the line segment $\overline{cp}$ is contained in S. A set $S \subset \mathbb{R}^d$ is a *pseudotriangle* $\iff$ it is a simple polygon and has exactly three convex vertices (see Figure 1).

In the following we consider subsets of $\mathbb{R}^d$. Prove or disprove:

a) Every star-shaped set is convex.

b) Every convex set is star-shaped.

c) The intersection of two convex sets is convex.

d) The union of two convex sets is convex.

e) The intersection of two star-shaped sets is star-shaped.

f) The intersection of a convex set with a star-shaped set is star-shaped.

g) Every pseudotriangle is star-shaped.

see reverse

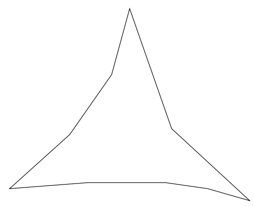

Figure 1: A pseudotriangle

Exercise 2

Consider three points $p, q, r \in \mathbb{R}^2$, given by their Cartesian coordinates $p = (p_x, p_y)$, $q = (q_x, q_y)$ and $r = (r_x, r_y)$. Show: the sign of the determinant

$$\begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$$

determines if r lies to the right, to the left or on the directed line through p and q.

Exercise 3

Let $P \subset \mathbb{R}^2$ be a convex polygon, given as an array $\texttt{p[0]}\dots\texttt{p[n]}$ of its $n+1$ vertices in counter clockwise order.

(a) Describe an algorithm with running time $O(\log(n))$, which determines whether a point q lies inside, outside or on the boundary of P.

(b) Describe an algorithm with running time $O(\log(n))$, which finds a (right) tangent to P from a query point q outside P (i.e. you should find a vertex $p[i]$, s.t. whole P is contained in a (left) halfplane determined by the line $qp[i]$).

*Exercise 4 (Caratheodory's Theorem)

Let $P = \{p_1, \dots, p_n\}$ be a set of $n \geq d+1$ points in $\mathbb{R}^d$ and let $q \in \text{conv}(P)$ be another point. Prove that there exists a subset $P' \subseteq P$ consisting of $d+1$ points such that $q \in \text{conv}(P')$.

*Exercise 5

Prove or disprove: The convex hull of a closed subset of $\mathbb{R}^d$ is closed.