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Computational Geometry — Exercise Set 5 — HS09

URL: http://www.ti.inf.ethz.ch/ew/courses/CG09/

Exercise 1

For a sequence of n pairwise distinct numbers $y_1, \dots, y_n$ consider the sequence of pairs $(\min\{y_1, \dots, y_i\}, \max\{y_1, \dots, y_i\})_{i=0,1,\dots,n}$ ($\min \emptyset := +\infty, \max \emptyset := -\infty$). How often do these pairs change in expectation if the sequence is permuted randomly, each permutation appearing with the same probability? Determine the expected value.

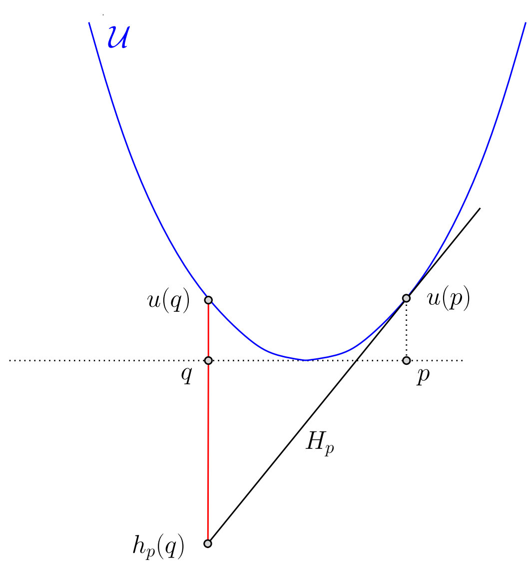
Exercise 2

Consider the unit paraboloid $\mathcal{U} : z = x^2 + y^2$ in $\mathbb{R}^3$ and let $u : p = (p_x, p_y, 0) \mapsto (p_x, p_y, {p_x}^2 + {p_y}^2)$ be the orthogonal projection of the x/y-plane onto $\mathcal{U}$. What is the equation for the tangent plane H_p to $\mathcal{U}$ in $u(p)$?

Let p and q be two points in the x/y-plane and $h_p : \mathbb{R}^3 \to H_p$ the orthogonal projection (i.e. in z-direction) of the x/y-plane onto H_p. Show:

$$\|u(q) - h_p(q)\| = \|p - q\|^2 \ .$$

Here is an illustration:



See reverse

Exercise 3

This exercise is about an application from *Computational Biology*:
You are given a set of disks $P = \{a_1, .., a_n\}$ in $\mathbb{R}^2$, all with the same radius $r_a > 0$. Each of these disks represents an atom of a protein. A water molecule is represented by a disc with radius $r_w > r_a$. A water molecule cannot intersect the interior of any protein atom, but it can be tangent to one. We say that an atom $a_i \in P$ is *solvent-accessible* if there exists a placement of a water molecule such that it is tangent to a_i and does not intersect the interior of any other atom in P. Given P, find an $O(n \log n)$ time algorithm which determines all solvent-inaccessible molecules of P.