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Computational Geometry Exercise Set 8 HS09

URL: http://www.ti.inf.ethz.ch/ew/courses/CG09/

Exercise 1

Recall the definition of the non-vertical geometric duality transform. To a point $a \in \mathbb{R}^2$ it assigns the line $a^* := \{x \in \mathbb{R}^2 \mid x_2 = a_1x_1 - a_2\}$ and to a non-vertical line l, which can be uniquely written in a form $l = \{x \in \mathbb{R}^d \mid x_2 = a_1x_1 - a_2\}$, it assigns a point $l^* := a \in \mathbb{R}^2$.

Describe the image of the following point sets under this mapping

a) a half plane

b) $k \geq 3$ colinear points

c) a line segment

d) the boundary points of the upper convex hull of a finite point set.

Exercise 2

Let L be a set of n lines in $\mathbb{R}^2$ no three of which pass through a common point. Suppose that all lines from $P \subseteq L$ are parallel to each other, no two lines from $L \setminus P$ are parallel to each other, and no line from $L \setminus P$ is parallel to those from P. Determine the number of vertices, edges, and faces of the arrangement $\mathcal{A}(L)$ in terms of n and $k := |P|$.

Exercise 3

For an arrangement $\mathcal{A}$ of a set of n lines in $\mathbb{R}^2$, let $\mathcal{F} := \bigcup_{C \text{ is cell of } \mathcal{A}} \overline{C}$ denote the union of the closure of all bounded cells. Show that the complexity (number of vertices and edges of the arrangement lying on the boundary) of $\mathcal{F}$ is $O(n)$.

Exercise 4

Given a set of lines in the plane with no three intersecting in a common point, form a graph G whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$. (χ is the chromatic number of the graph)