

Computational Geometry**Exercise Set 9****HS09**URL: <http://www.ti.inf.ethz.ch/ew/courses/CG09/>**Exercise 1**

Prove or disprove the following statement: Given three finite sets A, B, C of points in the plane, there is always a circle or a line that bisects A, B and C simultaneously (that is, no more than half of the points of each set are inside or outside the circle or on either side of the line, respectively).

Exercise 2

The 3-sum' problem is defined as follows: given 3 sets S_1, S_2, S_3 of n integers each, are there $a_1 \in S_1, a_2 \in S_2, a_3 \in S_3$ such that $a_1 + a_2 + a_3 = 0$? Prove that the 3-sum' problem and the 3-sum problem as defined in the lecture (there we had $S_1 = S_2 = S_3$) are equivalent, more precisely, that they are reducible to each other in subquadratic time. (Given an instance I of one of them, produce an instance I' of the other with size $O(n)$ such that I has a solution if and only if I' has a solution.)

Exercise 3

Describe an $O(n^2)$ time algorithm that given n points in the plane finds a subset of five points that form a strictly convex empty pentagon (or reports that there are none if that is the case). More precisely, the algorithm should output 5 points P' such that $\text{conv}(P') \cap P = P'$ and the vertices of $\text{conv}(P')$ are exactly the points P' .

Hint: Start with a point $p \in P$ that is extremal in one direction and try to find out whether there is a solution P' containing p .

Remark: It was shown by Heiko Harborth in 1978 that for every set of ten or more points in general position, some five of them form a strictly convex empty pentagon.