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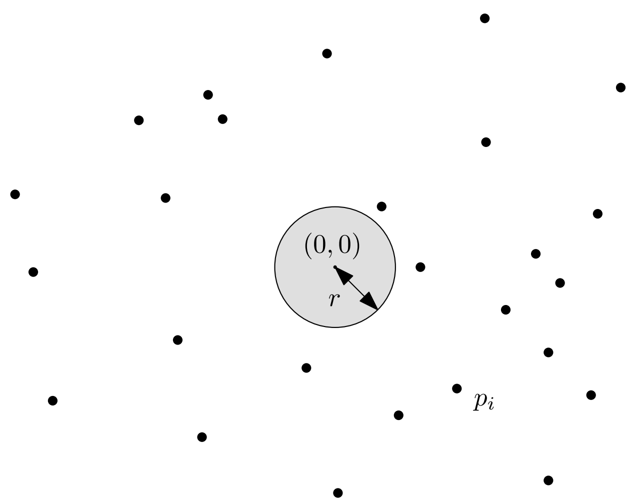
Computational Geometry — Homework 2 — HS09

URL: http://www.ti.inf.ethz.ch/ew/courses/CG09/

Exercise 1

Let $\{p_1, p_2, \ldots, p_n\}$ be a set of points in the plane, which we call *obstacles*. Imagine there is a disk of radius r which can be moved around between these obstacles but isn't allowed to intersect them (touching is ok). Is it possible to move the disk out of these obstacles if it is centered at the origin at first? More formally, the question is whether there is a (continuous) path $\gamma : [0,1] \longrightarrow \mathbb{R}^2$ with $\gamma(0) = (0,0)$, $\|\gamma(1)\| \geq \max\{\|p_1\|, \ldots, \|p_n\|\}$ such that at any time $t \in [0,1]$ and for any $1 \leq i \leq n$, $\|\gamma(t) - p_i\| \geq r$. Describe an algorithm to decide this question and to construct such a path if one exists, given arbitrary points $\{p_1, p_2, \ldots, p_n\}$ and a radius r. Argue why your algorithm is correct and analyze its running time.



Hints and Details: It is a good idea to compute the Voronoi diagramm of the point set first. (You don't have to describe how to do that, we assume it to be known.) Then you should argue why it makes sense to consider paths that run along the edges of the diagram and how you can get there first.

Exercise 2

Let H be a set with n elements and $f : 2^H \to \mathbb{R}$ a function that maps subsets of H to real numbers. We say that $h \in H$ *violates* $G \subseteq H$ if $f(G \cup \{h\}) \neq f(G)$ (it follows that $h \notin G$). We also say that $h \in H$ is *extreme* in G if $f(G \setminus \{h\}) \neq f(G)$ (it follows that $h \in G$).

Now we define two random variables $V_r, X_r : \binom{H}{r} \to \mathbb{R}$ where V_r maps an r-element set R to the number of elements that violate R, and X_r maps an r-element set R to the number of extreme elements in R.

a) Prove the following equality for $0 \leq r < n$:

$$\frac{E(V_r)}{n-r} = \frac{E(X_{r+1})}{r+1}.$$

Thus, there is a simple relation between the expected number of violating elements of a random r-element set and the expected number of extreme elements of a random $(r+1)$-element set.

b) Use the result from (a) to answer the following question: If you choose a random r-element set of $\{1, 2, \dots, n\}$, how many elements of $\{1, 2, \dots, n\}$ are expected to be larger than the maximum of the r chosen numbers?

Exercise 3

Let $P = \{p_1, p_2, \dots, p_n\}$ be a set of points in the plane (not all of them on the same line).

a) Prove that there is a triangulation of the vertices of the convex hull $\mathrm{conv}(P)$ with the property that the circumcircle of every triangle contains the whole set P.

b) Given a query point q we want to find the point $p_i \in P$ that is furthest from q. Following the locus approach, subdivide the plane into regions of equal answers. Describe how you can use a triangulation as defined in a) to find this subdivision.

Due date: 29.10.2008, 13h00