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Graph Theory — *Problem Set 7*

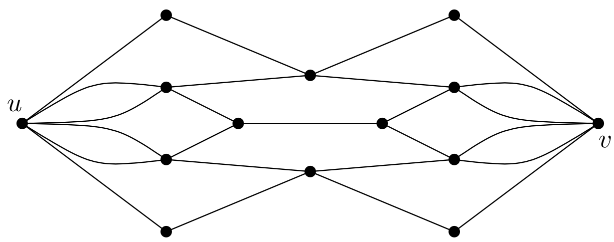
Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/GT04/`

Due Date: December 8, 2004 at the lecture

Exercise 7.1

(Exercise 4.2.1, 4.3.2 in the Textbook)

Determine $\kappa(u,v)$ and $\kappa'(u,v)$ in the graph drawn left below. (Hint: Use the dual problems to give short proofs of optimality.)



Exercise 7.2

(Exercise 4.2.8 in the Textbook)

Prove that a simple graph G is 2-connected if and only if for every ordered triple (x,y,z) of distinct vertices, G has an x,z-path through y. (Chien [1968])

Exercise 7.3

(Exercise 4.1.9 in the Textbook)

For each choice of integers k,ℓ,m with $0 < k \le \ell \le m$, construct a simple graph G with $\kappa(G) = k$, $\kappa'(G) = \ell$, and $\delta(G) = m$. (Make sure that you also explain why those three parameters for the graph you constructed are not less than what you claim!) (Chartrand–Harary [1968])

Exercise 7.4

(Exercise 3.3.16 in the Textbook)

(!) Prove that every $(k-1)$-edge-connected k-regular graph of even order has a 1-factor.

Exercise 7.5

(Exercise 4.1.14 in the Textbook)

(!) Let G be a connected graph in which for every edge e there are cycles C_1 and C_2 containing e whose only common edge is e. Prove that G is 3-edge-connected. Use this to show that the Petersen graph is 3-edge-connected.

Exercise 7.6

(Exercise 4.1.23 in the Textbook)

(!) Let G be an r-connected graph of even order having no $K_{1,r+1}$ as an induced subgraph. Prove that G has a 1-factor. (Sumner [1974b])