

Institut für Theoretische Informatik  
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## Graph Theory

## Problem Set 10

**Course Webpage:** <http://www.ti.inf.ethz.ch/ew/courses/GT04/>

**Due Date:** January 12, 2005 at the lecture

### Exercise 10.1

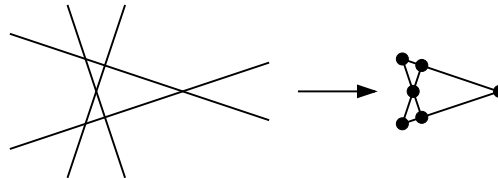
(Exercise 5.2.5 in the Textbook)

(-) Find a subdivision of  $K_4$  in the Grötzsch graph.

### Exercise 10.2

(Exercise 5.1.22 in the Textbook)

(!) Given a set of lines in the plane with no three meeting at a point, form a graph  $G$  whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that  $\chi(G) \leq 3$ . (H. Sachs)



### Exercise 10.3

(Exercise not in the Textbook)

(!) Prove that the complement of a bipartite graph is perfect. (This implies the weak perfect graph conjecture for bipartite graphs.)

### Exercise 10.4

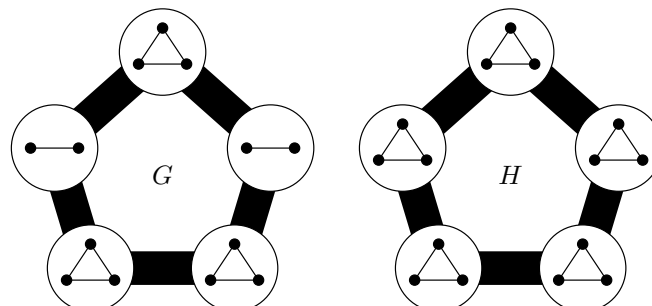
(Exercise 5.2.9 in the Textbook)

(!) Prove that if  $G$  is a color-critical graph, then the graph  $G'$  generated from  $G$  by applying Mycielski's construction is also color-critical.

### Exercise 10.5

(Exercise 5.2.40 in the Textbook)

Thick edges below indicate that every vertex in one circle is adjacent to every vertex in the other. Prove that  $\chi(G) = 7$  but  $G$  has no  $K_7$ -subdivision. Prove that  $\chi(H) = 8$  but  $H$  has no  $K_8$ -subdivision. (Catlin [1979])



### **Exercise 10.6**

(Exercise 5.1.25 in the Textbook)

(+) Let  $G$  be the **unit-distance graph** in the plane;  $V(G) = \mathbb{R}^2$ , and two points are adjacent if their Euclidean distance is 1 (this is an infinite graph). Prove that  $4 \leq \chi(G) \leq 7$ . (Hint: For the upper bound, present an explicit coloring by regions, paying attention to the boundaries.) (Hadwiger [1945, 1961], Moser–Moser [1961])

*Have a nice Christmas and a happy new year!*