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Graph Theory — Problem Set 2

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/GT04/`

Due Date: November 3, 2004 at the lecture

Exercise 2.1

(Exercise 1.2.11 in the Textbook)

(−) Prove or disprove: If G is an Eulerian graph with edges e, f that share a vertex, then G has an Eulerian circuit in which e, f appear consecutively.

Exercise 2.2

(Exercise 1.2.20 in the Textbook)

(!) Let v be a cut-vertex of a simple graph G. Prove that $\overline{G} - v$ is connected.

Exercise 2.3

(Exercise 1.2.26 in the Textbook)

(!) Prove that a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of $V(H)$.

Exercise 2.4

(Exercise not in the Textbook)

(!) Prove that every 5-regular graph contains a cycle of length at least 6.

Exercise 2.5

(Exercise 1.2.40 in the Textbook)

(!) Let P and Q be paths of maximum length in a connected graph G. Prove that P and Q have a common vertex.

Exercise 2.6

(Exercise not in the Textbook)

(+)

a) Prove that every graph with 10 vertices contains K_4 or $\overline{K_3}$. (Hint: remember Exercise 1.6 from Problem Set 1.)

b) Prove that the same statement is true even for 9 vertices.

c) Prove that the same statement is not true for 8 vertices.