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# Graph Theory — Problem Set 7

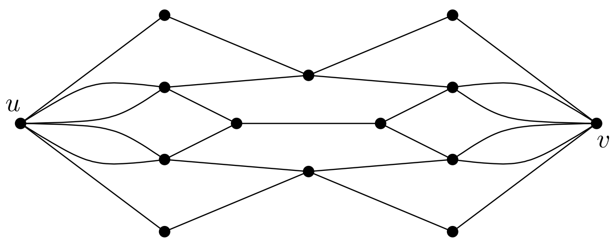
**Course Webpage:** `http://www.ti.inf.ethz.ch/ew/courses/GT04/`

**Due Date:** December 8, 2004 at the lecture

## Exercise 7.1

(Exercise 4.2.1, 4.3.2 in the Textbook)

Determine $\kappa(u, v)$ and $\kappa'(u, v)$ in the graph drawn left below. (Hint: Use the dual problems to give short proofs of optimality.)




## Exercise 7.2

(Exercise 4.2.8 in the Textbook)

Prove that a simple graph $G$ is 2-connected if and only if for every ordered triple $(x, y, z)$ of distinct vertices, $G$ has an $x, z$-path through $y$. (Chien [1968])

## Exercise 7.3

(Exercise 4.1.9 in the Textbook)

For each choice of integers $k, \ell, m$ with $0 < k \leq \ell \leq m$, construct a simple graph $G$ with $\kappa(G) = k$, $\kappa'(G) = \ell$, and $\delta(G) = m$. (Make sure that you also explain why those three parameters for the graph you constructed are not less than what you claim!) (Chartrand–Harary [1968])

## Exercise 7.4

(Exercise 3.3.16 in the Textbook)

(!) Prove that every $(k-1)$-edge-connected $k$-regular graph of even order has a 1-factor.

## Exercise 7.5

(Exercise 4.1.14 in the Textbook)

(!) Let $G$ be a connected graph in which for every edge $e$ there are cycles $C_1$ and $C_2$ containing $e$ whose only common edge is $e$. Prove that $G$ is 3-edge-connected. Use this to show that the Petersen graph is 3-edge-connected.

## Exercise 7.6

(Exercise 4.1.23 in the Textbook)

(!) Let $G$ be an $r$-connected graph of even order having no $K_{1,r+1}$ as an induced subgraph. Prove that $G$ has a 1-factor. (Sumner [1974b])