Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institut für Theoretische Informatik
Dr. Tibor Szabó and Miloš Stojaković

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Graph Theory — Problem Set 10

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/GT04/`

Due Date: January 12, 2005 at the lecture

Exercise 10.1

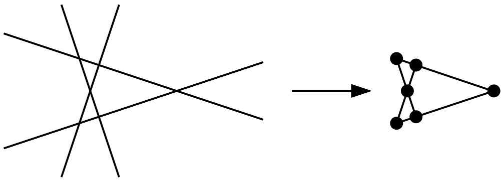
(Exercise 5.2.5 in the Textbook)

(−) Find a subdivision of K_4 in the Grötzsch graph.

Exercise 10.2

(Exercise 5.1.22 in the Textbook)

(!) Given a set of lines in the plane with no three meeting at a point, form a graph G whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$. (H. Sachs)



Exercise 10.3

(Exercise not in the Textbook)

(!) Prove that the complement of a bipartite graph is perfect. (This implies the weak perfect graph conjecture for bipartite graphs.)

Exercise 10.4

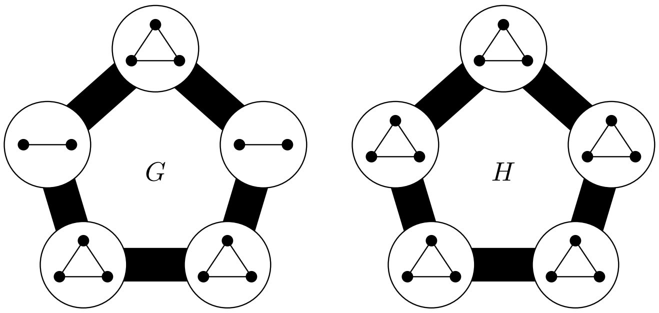
(Exercise 5.2.9 in the Textbook)

(!) Prove that if G is a color-critical graph, then the graph G' generated from G by applying Mycielski's construction is also color-critical.

Exercise 10.5

(Exercise 5.2.40 in the Textbook)

Thick edges below indicate that every vertex in one circle is adjacent to every vertex in the other. Prove that $\chi(G) = 7$ but G has no K_7-subdivision. Prove that $\chi(H) = 8$ but H has no K_8-subdivision. (Catlin [1979])



Exercise 10.6

(Exercise 5.1.25 in the Textbook)

(+) Let G be the **unit-distance graph** in the plane; $V(G) = \mathbb{R}^2$, and two points are adjacent if their Euclidean distance is 1 (this is an infinite graph). Prove that $4 \leq \chi(G) \leq 7$. (Hint: For the upper bound, present an explicit coloring by regions, paying attention to the boundaries.) (Hadwiger [1945, 1961], Moser–Moser [1961])

Have a nice Christmas and a happy new year!