ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

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Informatik für Mathematiker und Physiker HS12

Exercise Sheet 4

Submission deadline: 3.15pm - Tuesday 16th October, 2012

`Course URL`: http://www.ti.inf.ethz.ch/ew/Lehre/Info1_12/

Note: For the tasks Assignment 1 and Challenge 1, you are *not* allowed to use any other libraries than `<iostream>` and `<IFM/Integer.h>` and only the commands you have seen in the lectures until now.

Assignment 1 - (2 points)

The integer square root of a given *positive* integer n is defined as $\lfloor\sqrt{n}\rfloor$. The `<IFM/Integer.h>` library does not provide a method for getting the integer square root for a given number.

Write a program `integerSqrt.cpp` that reads from the standard input a non-negative number n of the type `ifm::integer` and outputs its integer square root.

Challenge 1 - (4 points)

Write a program `integerSqrtChallenge.cpp` that reads from the standard input a non-negative number n of the type `ifm::integer` and outputs its integer square root. The program should be capable of calculating the integer square root of a 100-digit integer within 1 second.

Assignment 2 - (4 points)

Write a program `perfect.cpp` to test whether a given natural number n is perfect. A number $n \in \mathbb{N}$ is called *perfect* if and only if it is equal to the sum of its proper divisors, that is, $n = \sum_{k \in \mathbb{N},\, s.t.\, k<n \,\wedge\, k|n} k$. For example, $28 = 1{+}2{+}4{+}7{+}14$ is perfect, while $12 < 1{+}2{+}3{+}4{+}6$ is not.

Extend the program to find all perfect numbers between 1 and n. How many perfect numbers exist in the range $[1, 50000]$?

Assignment 3 - (5 points)

The number π can be defined through various infinite sums. Here are two of them.

$$\begin{aligned}\frac{\pi}{4} &= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \\ \frac{\pi}{2} &= 1 + \frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7} + \cdots\end{aligned}$$

Write a program for computing an approximation of π, based on these formulas. Which formula is better for that purpose?

Assignment 4 - (5 points)

Noah is the owner of a well-established shop in Wengen and has his regular customers. He estimates that with his regular customers, he makes m CHF net profit a year. He wants to put his profit into Wengen Dorf Bank, that promises $p\%$ interest on the money on his account every year. Noah wants to know how much money he would have on his account in n years, if he deposits m CHF every year onto the account under $p\%$ interest, assuming that at the begining, the account was empty.

Write a program `interests.cpp` that reads m, n and p from the standard input and outputs the the amount of money that is at Noah's account after he deposits m CHF for n years on the account with $p\%$ interest rate.

The example inputs and outputs are:

```
The yearly amout m =? 1
The yearly interest (in %) p =? 5
The number of years n =? 1
The total amount on the account is 1.05

The yearly amout m =? 1
The yearly interest (in %) p =? 5
The number of years n =? 2
The total amount on the account is 2.1525
```

Challenge 2 - (4 points)

The largest Mersenne prime known as of September 2009 is

$$2^{43,112,609} - 1$$

Write a program `famous_last_digits.cpp` that computes and outputs the last 10 decimal digits of the above Mersenne prime!