Exam — Informatik D-MATH/D-PHYS
24. 1. 2014 09:00–11:00

Prof. Bernd Gärtner

Examinee:

Last name: .....

First name: .....

Stud. no.: .....

With my signature I attest, that I was able to sit the exam under regular conditions and that I read and understood the general remarks.

Signature: .....

General remarks:

1. Verify the completeness of the exam given to you (four two-sided sheets with 6 assignments in total and one empty sheet for notes)! **Fill in the title sheet, i.e., state your name and your student number, in a legible form.**
2. Allowed utilities: **None, except dictionaries.**
3. Cheating or attempts to cheat lead to immediate disqualification and may entail legal consequences.
4. **Write your solutions directly on the assignment sheets!** Only one solution attempt per assignment will be considered. **Hint:** Prepare your solutions on separate sheets and transfer only the final solutions to the assignment sheet. Clearly cross out solution attempts that you do not want to be considered. If the space provided on the assignment sheets is insufficient for your solution attempts, use separate sheets and label them with the assignment numbers and your name.
5. If you want to submit early, hand all relevant documents over to one of the invigilators before leaving the room.
6. **After 10:45, it is no longer possible to submit early. Please remain seated until the exam ends and until your documents have been collected by the invigilators.**
7. The exam's pass mark is 60 out of 120 points. **Good luck!**

1	2	3	4	5	6	$\sum$

Assignment 1. (18 Points) For each of the following 6 expressions, provide C++ type and value! No points will be awarded for intermediate steps of the evaluation. Assume that x has type **int** and value 3 *at the beginning of each subtask.*

Expression	Type	Value
`0 < 1 && 1 != 1 || ++x > 3`		
`12.0 / 3u / 2u`		
`2 + x++`		
`1.4e3 * 0.2e-2`		
`2014 % 4 + x % 2`		
`x / 2 + 9.0f / 6`		

Assignment 2. (15 Points) For each of the following three code fragments, write down the sequence of numbers that it outputs!

---

a)
```
for (int i=0; i<100; i*=2)
  std::cout << ++i << " ";
```

Output:

---

b)
```
int i=0;
int j=5;
while (i != j)
  std::cout << (i+=2) - j++ << " ";
```

Output:

---

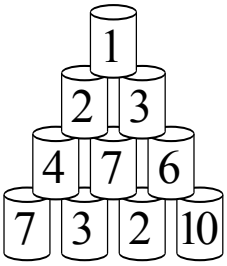
c)
```
void f(unsigned int x) {
  if (x == 0)
    std::cout << "-";
  else {
    std::cout << "*";
    f(x-1);
    std::cout << "*";
  }
}
f(5);
```

Output:

Assignment 3. (23 Points) In the game of can knockdown, cans are stacked on top of each other like in a pyramid. Each can has a certain value. This value is stored in a two-dimensional array. The first index of the array describes the row, the second index the position (column) within this row. We count rows from top to bottom, starting at 0, and columns from left to right, starting at 0. In the example below, the can in row 2, at position 0 has value 4 (`D[2][0] == 4`).

If a can gets hit, it will fall. All cans that are standing directly above another falling can will fall too. For example, if the can with value 7 in the middle of row 2 gets hit, the cans with values 7, 2, 3, and 1 will fall.

```
int D[4][4] = {{1, 0, 0, 0},
               {2, 3, 0, 0},
               {4, 7, 6, 0},
               {7, 3, 2, 10}};
```



The following is the skeleton for a function `hit`, that computes the sum of the values of all fallen cans, if can at position `c` in row `r` was hit. Complete the skeleton to a correct function by providing the four expressions `expr1`, `expr2`, `expr3` and `expr4`! You can assume that you can access the array `D` directly within the function, that is, `D` is a global variable.

```
// PRE:  the array D has size at least (n+1) x (n+1) (n = max(r, c)),
//       D[r][c] denotes the value of the can in row r, column c
//       where c <= r
// POST: returns the sum of the values of all cans that fall
//       if the can at position/column c in row r gets hit
int hit(int r, int c)
{
  if (r == 0)
    return expr1;
  if (c == 0)
    return expr2;
  if (c == r)
    return expr3;
  return expr4;
}
```

```
// PRE:  the array D has size at least (n+1) x (n+1) (n = max(r, c)),
//       D[r][c] denotes the value of the can in row r, column c
//       where c <= r
// POST: returns the sum of the values of all cans that fall
//       if the can at position/column c in row r gets hit
int hit(int r, int c)
{
  if (r == 0)
    return expr1;
  if (c == 0)
    return expr2;
  if (c == r)
    return expr3;
  return expr4;
}
```

---

expr1:

---

expr2:

---

expr3:

---

expr4:

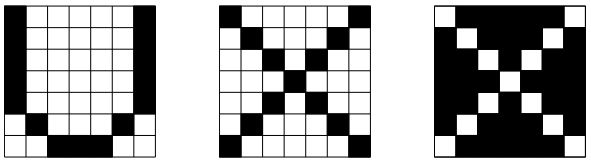
Assignment 4. (24 Points) Your task is to provide an implementation of a function that computes a floating-point approximation of the exponential function

$$e^x := \sum_{i=0}^{\infty} \frac{x^i}{i!},$$

for a given real number x. The implementation should be based on this definition of e^x as an infinite series. To obtain an approximation of e^x, the function should sum up initial terms of this series (in double precision floating-point arithmetic) *until* adding the current term does not change the value obtained so far anymore. You are *not* allowed to use any external libraries. The *total* number of statements in your solution must not be larger than the number of lines provided.

```
// POST: returns e^x
double exp (double x)
{
                                                    // 1
                                                    // 2
                                                    // 3
                                                    // 4
                                                    // 5
                                                    // 6
                                                    // 7
                                                    // 8
                                                    // 9
                                                    // 10
                                                    // 11
}
```

Assignment 5. (14 / 12 Points) Consider the following partial declaration of a class for representing black & white images of 7×7 pixels. Here are three examples of such images.



← Image for part b) of the assignment

```
// a class for 7x7 black-and-white pixel images
class image {
private:
  // bitmap[r][c] = true if and only if the pixel
  // in row r and column c is black
  bool bitmap[7][7];

public:
  // POST: *this is the empty image (all pixels are white)
  image ();

  // PRE: 0 <= r,c < 7
  // POST: the pixel in row r and column c of *this is set to black
  void set_pixel (int r, int c);

  // POST: the inverted image of *this is returned (black pixels
  //       become white and white pixels become black)
  image operator! () const;
};
```

a) Provide correct definitions of the three public member functions! The *total* number of statements in your solution must not be larger than the number of lines provided.

```
image::image()
{
                                                    // 1
                                                    // 2
                                                    // 3
}
```

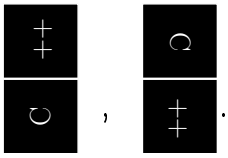
```
void image::set_pixel (int r, int c)
{
                                                    // 1
}

image image::operator! () const
{
                                                    // 1
                                                    // 2
                                                    // 3
                                                    // 4
                                                    // 5
                                                    // 6
}
```

b) Assuming that there is an output operator for the class `image`, provide a definition of the main function that outputs the rightmost image on the previous page!

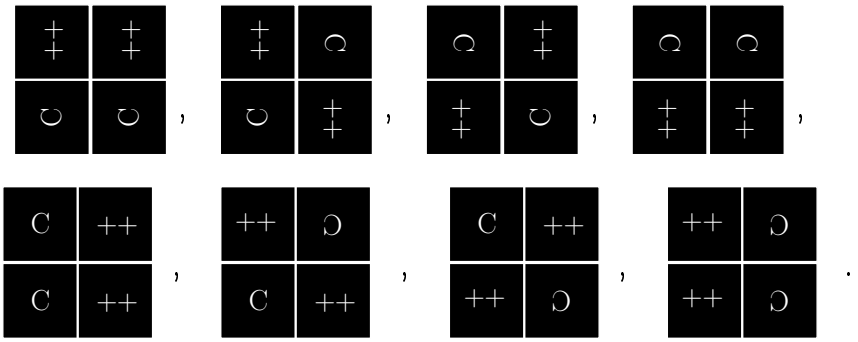
```
int main()
{
                                                    // 1
                                                    // 2
                                                    // 3
                                                    // 4
                                                    // 5
                                                    // 6
                                                    // 7
                                                    // 8
}
```

Assignment 6. (10 /4 Points) Let n be a positive natural number. A board of size $2 \times n$ shall be filled with domino tiles of size 1×2, where each domino tile looks like this:

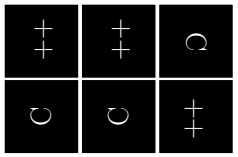
Let $T(n)$ be the number of different possibilities of filling the board. We obtain for example $T(1) = 2$:



For $n = 2$ we get $T(2) = 8$:



For $n = 3$, we just show one of the many possibilities.



a) Give a recursive formula for $T(n)$ if $n \geq 3$. You do not have to prove that the formula is correct.

$$T(n) = \qquad\qquad , \text{ if } n \geq 3.$$

b) Compute, using a) or any other means, the value $T(4)$.

$$T(4) =$$