ETH Zürich **Metric Embeddings**
Institute of Theoretical Computer Science **Spring 2010**

Problem Set 2

(to be handed in March 22 in class)

1. (a) Determine the distortion of the identity mapping $(\mathbb{R}^d, \|.\|_2) \to (\mathbb{R}^d, \|.\|_1)$.

   (b) Determine the distortion of the identity mapping $(\mathbb{R}^d, \|.\|_1) \to (\mathbb{R}^d, \|.\|_\infty)$. Find a mapping between the same spaces with a smaller distortion. (How small can you make it?)

2. True or false? There is a function $\varphi(n)$ with $\lim_{n\to\infty} \frac{\varphi(n)}{n} = 0$ such that every n-vertex tree (shortest-path metric, unit-length edges) can be embedded into $\mathbb{R}^1$ with distortion at most $\varphi(n)$.

3. (a) Prove that every ℓ_1 metric ρ on a finite set X can be expressed as a nonnegative linear combination of cut metrics on X. (That is, there exist cut metrics ν_1, ν_2, …, ν_k on X and nonnegative real α_1, …, α_k with $\rho(x, y) = \sum_{i=1}^{k} \alpha_i \nu_i(x, y)$ for all $x, y \in X$.)

   (b) Show that every embedding of $K_{2,3}$ into ℓ_1 has distortion at least 4/3. Show that this bound is tight.

4. Show that every n-vertex unweighted graph can be embedded into $\mathbb{R}^1$ with distortion $O(n)$. (Hint: Try first solving the problem for trees.)

Food for thought (will be solved in class next week)

1. Show that the complete binary tree of height m can be embedded into ℓ_2 with distortion $O(\sqrt{\log m})$.