Exercise 1. Decide, which pairs of the following spaces are homeomorphic (include, not necessarily formal, proofs): \((0,1), \[0,1\], S^1, \mathbb{R}, \) and the trefoil knot \(K\) drawn on the figure.

![Trefoil Knot](image)

Exercise 2. Let \(X\) and \(Y\) be topological spaces, \(f: X \rightarrow Y\) a continuous mapping, and \(M, N \subseteq X\). Prove or disprove the following statements:

(a) If \(M\) is closed then \(f(M)\) is closed.
(b) If \(M\) is open then \(f(M)\) is open.
(c) If \(M\) is connected then \(f(M)\) is connected.
(d) If \(M\) is unconnected then \(f(M)\) is unconnected.
(e) If \(M\) is closed and \(N\) compact then \(M \cap N\) is compact.

Exercise 3. Let \(T\) be a torus. Show that there is no continuous injection \(f: T \rightarrow S^2\). Hint: Show that the graphs \(K_5\) and \(K_{3,3}\) embed into \(T\) and use the Kuratowski theorem.

Exercise 4. (a) Find a triangulation of the cube \([0,1]^3\) using just five 3-simplices. (It is sufficient just to draw a figure.) Hint: Try to find a regular tetrahedron spanned by some of the vertices of the cube.
(b) Prove that every triangulation of the cube \([0,1]^3\) uses at least five 3-simplices. Hint: Derive that each simplex covers at most \(\frac{1}{4}\) of the surface of the cube and in such case it is unique (up to isometries of the cube).

Exercise 5. For positive integer \(n\) prove or disprove the following statement: Whenever \(A_1, A_2, \ldots, A_{n+1}\) is a cover of \(S^n\) by \(n+1\) sets, each of which can be obtained from open sets by finitely many set-theoretic operations (union, intersection, difference), then there is \(i\) with \((A_i) \cap (-A_i) \neq \emptyset\).