General rules for solving exercises

- When handing in your solutions, please write your exercise group on the front sheet:
  
  **Group A/B:** Wed 13–15 CAB G 56  
  **Group C:** Wed 16–18 CAB G 52

- This is a theory course, which means: if an exercise does not explicitly say “you do not need to prove your answer”, then a formal proof is always required.

The following exercises will be discussed in the exercise class on November 16, 2016. Please hand in your solutions not later than November 15.

**Exercise 1: Finding a Separating Line**

Let $R, B \subseteq \mathbb{R}^2$ be given finite sets (“red and blue points”). A *strictly separating line* is a line $\ell$ with the property that all red points lie strictly on one side of $\ell$, and all blue points strictly on the other side.

Formulate a linear program such that, given an optimal solution of your LP, you can decide if a separating line exists and, if so, compute one.

**Exercise 2: Fitting a Ball into a Convex Polytope**

Let $H_1, \ldots, H_m$ be halfspaces in $\mathbb{R}^n$ given by $H_i = \{x : a_i^T x \leq b_i\}$ where $a_i \in \mathbb{R}^n$ and $b_i \in \mathbb{R}$. We want to find the largest $n$-dimensional ball that is completely contained in the intersection $\bigcap_{i=1}^m H_i$, which is assumed to be non-empty.

Formulate a linear program with variables $c \in \mathbb{R}^n$ and $r \in \mathbb{R}$ whose optimal solution is the center point $c^*$ and radius $r^*$ of this largest ball.
Exercise 3: Linear Programs in Equational Form

Show that every linear program can also be converted into the following equational form:

\[
\text{maximize } c^T x \text{ subject to } Ax = b, \ x \geq 0.
\]

What is the maximum increase in the number of variables and in the number of constraints in such a transformation?

Exercise 4: Maximum Number of Vertices of 3-dimensional Convex Polytopes

Let \( P \) be a convex polytope which is defined as the intersection of \( n \) given closed half-spaces in \( \mathbb{R}^3 \). Show that the number of vertices of \( P \) is at most \( 2n - 4 \).

HINT: Use Euler's formula for plane graphs, \( v - e + f = 2 \).

Exercise 5: Certificates for Infeasibility of Systems of Linear Equations

(Preparation for chapter 6.5.)

Prove that a system \( Ax = b \) of linear equations is unsolvable if and only if there is \( y \) with \( A^T y = 0 \) and \( b^T y = 1 \).