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Algorithms, Probability, and Computing — Exercises KW51 — HS16

The following exercises will be discussed in the exercise class on December 21, 2016. Please hand in your solutions not later than December 20.

Exercise 1: MIS via Network Decomposition

Explain how, given a (C, D)-network decomposition, a maximal independent set can be computed in $O(CD)$ rounds.

Exercise 2: Near-Optimality of Theorem 8.27

It is known that there are n-node graphs that have girth $\Omega(\log n/\log\log n)$ and chromatic number $\Omega(\log n)$. Use this fact to argue that on these graphs, an $(o(\log n), o(\log n/\log\log n)))$ network decomposition does not exist.

Exercise 3: Diameter Orderings

Given an n-node undirected graph $G = (V, E)$, a $d(n)$-diameter ordering of G is a one-to-one labeling $f : V \to [n]$ of vertices such that for any path $v_1, \dots, v_p$ on which the labels $f(v_i)$ are monotonically increasing, any two nodes v_i, v_j have $\mathrm{dist}_G(v_i, v_j) \le d(n)$.

Use the network decomposition of theorem 8.27 to argue that each n-node graph has an $O(\log^2 n)$-diameter ordering.

Exercise 4: Ruling Sets

Consider the following 1-round randomized algorithm: Each node v picks a random real number $r_v \in [0, 1]$ and sends it to its neighbors. Then v joins a set S if its random number is a local minimum, that is, if $r_v < r_u$ for all neighbors u of v.

Prove that, with high probability, the set S is a $(2, O(\log n))$-ruling set.