In-Class Exercise 1: Bounding a Sequence

Suppose that a sequence \( \{x_n\}_{n \geq 1} \) with \( x_1 > 0 \) satisfies

\[
x_n = \sqrt{n} + 2 \sum_{i=1}^{n-1} x_i
\]

for \( n \geq 2 \).

Prove or disprove: \( x_n = O(2^n) \).

In-Class Exercise 2: Random Permutations

For a permutation \( \pi \) on the keys \( \{1..n\} \), let \( T_\pi \) be the search tree that we obtain from inserting all \( n \) keys, one after the other, in the order given by \( \pi \).

Prove: If \( \pi \) is drawn uniformly at random, then \( T_\pi \) has the same distribution as \( \tilde{\mathcal{B}}[n] \) from the lecture.