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Algorithms, Probability, and Computing **In-Class Exercise KW48** **HS16**

Exercise 1: Strong Duality (6.6)

Find an example of a specific linear program (P) for each of the cases in Theorem 6.6.

Exercise 2: The Subtour LP (6.10, 6.11, 6.12)

Recall the definition of the Subtour LP:

$$\text{minimize } c^{\mathsf{T}}x \text{ subject to} \qquad \begin{aligned} &\sum_{e\in\delta(v)} x_e = 2 \text{ for all } v \in V \\ &\sum_{e\in\delta(S)} x_e \geq 2 \text{ for all } S \subseteq V \text{ with } \emptyset \neq S \neq V \\ &1 \geq x_e \geq 0 \text{ for all } e \in E. \end{aligned}$$

(i) Show that a graph $G = (V, E)$ is connected if and only if $\delta(S) \neq \emptyset$ for all $S \subseteq V$, $\emptyset \neq S \neq V$. Prove this using the most basic definition of connectedness: A graph is connected if for any two vertices $v, w \in V$ there is a path from v to w.

(ii) Give a graph G for which the subtour LP is infeasible.

(iii) Give a graph G where the subtour LP is feasible but there is no feasible integer solution.

(iv) Assume $|V| \geq 3$. Show that the constraints "$1 \geq x_e$" are redundant in the Subtour LP, i. e. every point $x \in \mathbf{R}^E$ that is feasible w.r.t. all other constraints does satisfy $1 \geq x_e$ for all $e \in E$.

Exercise 3: The Loose Spanning Tree LP (6.15)

In the lecture we have considered the graph $G_{k,\ell}$ which consists of a k-clique with an attached path of length ℓ. We have specified costs $c_e := 0$ for all edges of the k-clique and $c_e := \gamma$ for all edges of the ℓ-path ($\gamma > 0$).

Show that the value of the Loose Spanning Tree LP for $G_{k,\ell}$ with $\ell = k(k-3)+4$ is *exactly* $\ell\gamma/2$.

Exercise 4: Loose and Tight Spanning Tree LP (6.16)

Show that every feasible point of the Tight Spanning Tree LP is feasible in the Loose Spanning Tree LP – without using theorem 6.11.

Exercise 5: Let's Relax (6.17)

Let $m \in \mathbf{N}$, $c \in \mathbf{R}^m$, $S \subseteq \mathbf{R}^m$ finite and $P := \text{conv}(S)$. Show that we have

$$\min_{x \in P} c^T x = \min_{x \in S} c^T x.$$