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Approximation Algorithms and Semidefinite Programming FS12
Exercise Set 1

Course Webpage: `http://www.ti.inf.ethz.ch/ew/lehre/ApproxSDP12/`
Due date: March 6, 2012.

Exercise 1 (Derandomization of `RandomizedMaxCut`)

[Exercise 1.1] Prove that there is a deterministic 0.5-approximation algorithm for the MaxCut problem.

Exercise 2 (Improving `RandomizedMaxCut`)

In this question we ask you to come up with two (deterministic or randomized) approximation algorithms that slightly improve the simple `RandomizedMaxCut` algorithm from the lecture.
For a given graph $G = (V, E)$, let $n = |V|$ and $m = |E|$ denote the number of vertices and edges of G.

a) *[Exercise 1.2]* Prove that there is a $\left(\frac{1}{2} + \frac{1}{2m}\right)$-approximation algorithm for the MaxCut problem.

b) Prove that there is a $\left(\frac{1}{2} + \frac{1}{2n}\right)$-approximation algorithm for the MaxCut problem.

Hint. A deterministic algorithm for part b) might make use of the following Lemma (which you don't need to prove.)
Lemma. Let $G = (V, E)$ be a graph with $|V|$ even. A matching of size at least $|E|/(|V| - 1)$ can be computed in time $O(|V| + |E|)$.

Exercise 3 (Positive Semidefinite Matrices)

[Fact 2.2.1] Let $M \in \mathrm{SYM}_n$. Prove that the following statements are equivalent.

i) M is positive semidefinite, i.e., all the eigenvalues of M are nonnegative.

ii) $\mathbf{x}^T M \mathbf{x} \geq 0$ for all $\mathbf{x} \in \mathbb{R}^n$.

iii) There exists a matrix $U \in \mathbb{R}^{n \times n}$ such that $M = U^T U$.