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Approximation Algorithms and Semidefinite Programming FS12
Exercise Set 5

Course Webpage: `http://www.ti.inf.ethz.ch/ew/lehre/ApproxSDP12/`
Discussion: Mai 15

Exercise 1 (Hypercube solutions to (GW))

[Exercise 8.3] Suppose that for some graph G, the Goemans-Williamson vector program (GW) has an optimal solution whose vectors are all contained in the set $\{-\frac{1}{\sqrt{d}}, \frac{1}{\sqrt{d}}\}^d \subset \mathbb{R}^d$ (for some $d \leq n$). Prove that then the integrality gap equals 1; that is, there exists a cut in G whose number of edges equals the optimum of (GW).

Exercise 2 (Coloring 3-colorable graphs)

[Exercise 9.2] Suppose that an algorithm is given that, for every n-vertex 3-colorable graph with maximum degree Δ, finds an independent set of size at least $cn/\Delta^{1/3}$, where $c > 0$ is a constant. Show that using this algorithm, we can do the following:

a) We can color every n-vertex 3-colorable graph with $\tilde{O}(\Delta^{1/3})$ colors.

b) We can color every n-vertex 3-colorable graph with $\tilde{O}(n^{1/4})$ colors.

Exercise 3 (Frankl-Wilson inequality)

[Exercise 9.4] Use the polynomial method to prove the Frankl-Wilson inequality:

Let p be a prime, and let d and s be integers with $d > s \geq p$. Let $\mathcal{F}$ be a system of s-element subsets of $\{1, \ldots, d\}$ such that for every two distinct $A, B \in \mathcal{F}$, we have $|A \cap B| \not\equiv s \pmod{p}$. Then

$$|\mathcal{F}| \leq \sum_{i=0}^{p-1} \binom{d}{i}.$$