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Extremal Combinatorics — *SS 07*

Exercise Set 1

Exercise 1

Suppose $n \geq 4$ and let H be an n-uniform hypergraph, i.e. $H = (V, E)$ with $E \subseteq \binom{H}{n}$. Prove that if $|E| \leq \frac{4^{n-1}}{3^n}$, then there exists a coloring of the vertices of H by 4 colors so that in every edge all 4 colors are represented.

Exercise 2

Let $\{(A_i, B_i) \,|\, 1 \leq i \leq h\}$ be a family of pairs of subset of the set of integers of such that $|A_i| = k$ and $|B_i| = \ell$ for all i and $A_i \cap B_i = \emptyset$ and $A_i \cap B_j \neq \emptyset$ for all $i \neq j$. Prove that $h \leq \binom{k+\ell}{k}$.

Hint: Look at a random order π of $\bigcup_i (A_i \cup B_i)$ and consider the probabilities that all the elements of A_i preceed all those of B_i in this order.

Exercise 3

Study the construction of the projective plane $PG(q, 2)$ for q a prime tower (given on the extra sheet) and verify the axioms of a projective plane by showing that

(i) every pair of distinct lines is incident to a unique point,

(ii) every pair of distinct points is incident to a unique line,

(iii) there are four points such that no three of them are incident to a single line.