

Institut für Theoretische Informatik Dr. Tibor Szabó and Philipp Zumstein

## Extremal Combinatorics

Ecole polytechnique fédérale de Zurich Politecnico federale di Zurigo Swiss Federal Institute of Technology Zurich

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## SS 07 Exercise Set 10

## Exercise 1

We want to improve on the quadratic lower bound of the Ramsey number giving an explicit construction which will still fall short of the  $\sqrt{2}^k$  probabilistic lower bound, but will improve over the Turán graph.

For this purpose consider the following construction by Nagy (1972) of an infinite sequence of k-Ramsey graphs on  $\Theta(k^3)$  vertices.

Let G be the graph with  $V(G) = {\binom{[k]}{3}}$ , and we define two vertices A and B to be adjacent if and only if  $|A \cap B| = 1$ .

Prove *combinatorial* (without using the linear algebra method!) that this graph neither contains a clique of order k + 1 nor an independent set of order k + 1.