# Extremal Combinatorics

# SS 07
# Exercise Set 10

## Exercise 1

We want to improve on the quadratic lower bound of the Ramsey number giving an explicit construction which will still fall short of the $\sqrt{2}^k$ probabilistic lower bound, but will improve over the Turán graph.

For this purpose consider the following construction by Nagy (1972) of an infinite sequence of $k$-Ramsey graphs on $\Theta(k^3)$ vertices.

Let $G$ be the graph with $V(G) = \binom{[k]}{3}$, and we define two vertices $A$ and $B$ to be adjacent if and only if $|A \cap B| = 1$.

Prove *combinatorial* (without using the linear algebra method!) that this graph neither contains a clique of order $k + 1$ nor an independent set of order $k + 1$.