Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institut für Theoretische Informatik
Dr. Tibor Szabó and Philipp Zumstein

27.04.07

Extremal Combinatorics — *SS 07*

Exercise Set 4

Exercise 1

A natural thought to extend the idea of the Brown graph to $K_{4,4}$- or $K_{4,1000}$-avoiding dense graphs is the following. Instead of three dimensions let us take four, i.e. our vertex set is $\mathbb{F}_p^4$. Let the neighborhood of a vertex x be determined by a four-dimensional sphere around it, in particular y is adjacent to x if $\sum_{i=1}^4 (y_i - x_i)^2 = 1$. According to Theorem 2.6, our graph has roughly $cn^{7/4}$ edges — the conjectured truth. Prove, however, that this graph contains a $K_{p,p}$.

Also show that even taking a higher degree surface of the form $\sum_{i=1}^4 (y_i - x_i)^{1000} = 1$ as the neighborhood of x instead of the sphere would not help us. (Note that Theorem 2.6 ensures that this graph as well has roughly the correct number $cn^{7/4}$ of edges.)

Exercise 2

Prove that in Brown graph roughly half of the triples has two common neighbors and the other half has none.

Exercise 3

Define the graph $G = (\mathbb{F}_p^4, E)$ by

$$\{(a, b, c, d), (a', b', c', d')\} \in E \text{ iff } (a + a')(b + b')(c + c')(d + d') = 1 \text{ and } (a, b, c, d) \neq (a', b', c', d')\,.$$

Prove that G contains a $K_{n^{1/4}, n^{1/4}}$ where $n = |\mathbb{F}_p^4|$.