Extremal Combinatorics   *SS 07*
Exercise Set 5

Exercise 1

The k-color Ramsey Number $R_k(G)$ is the largest integer m for which one can color the edges of K_m with k colors such that there is no monochromatic copy of G.

Show that $R_k(K_{3,3}) = (1 + o(1))k^3$.

Exercise 2

Using Füredi's idea together with the projective norm-graphs prove that for any fixed t

$$\lim_{s \to \infty} (\liminf_{n \to \infty} ex(n, K_{t,s})n^{-(2-1/t)}) = \infty.$$

In particular for $t = 3$, give a construction such that the known upper bound of $ex(n, K_{3,s})$ comes within a factor of $2^{-\frac{1}{3}} + o(1)$ of the lower bound for every $s \geq 3, s = 2r^2 + 1, r \in \mathbb{Z}$.