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Extremal Combinatorics — *SS 07*

Exercise Set 8

Exercise 1

Let $q \equiv 1 \pmod 4$ be a prime power, then -1 is a quadratic residue in the field $\mathbb{F}_q$. Now we define the *Paley graph* P_q as follows: $V(P_q) = \mathbb{F}_q$ and two vertices x and y are adjacent if and only if $x-y \in QR(q)$. Note that this relation is symmetric since we have chosen q in such a way that -1 is a quadratic residue, hence the graph is well-defined.

Prove that the Paley graph P_{17} neither contains a clique of order 4 nor an independent set of order 4.

Exercise 2

Prove that if

$$k = (4\log n)^{2^{\binom{n}{2}}};$$

then

$$N = n^{2^{\binom{n}{2}}} = k^{\Omega\left(\frac{\log\log\log k}{\log\log\log\log k}\right)}.$$